Devoir #6 MAT 3572 (2015) M. ALVO

Avr 14

1. #8 p271 a) $c\int_{-y}^{y}(y^2-x^2)e^{-x}dx = \frac{4}{3}\cdot c y^3 e^{-y} \qquad 0<y<\infty$

(3) $1 = \int_0^\infty f_Y(y)\,dy = \int_0^\infty \frac{4}{3} c y^3 e^{-y} dy = \frac{4}{3}c \cdot \Gamma(4) = \frac{4c}{3}\cdot 3! = 8c$

$\Rightarrow c = 1/8$

b) $f_X(x) = \frac{1}{8}\int_{|x|}^{\infty}(y^2-x^2)e^{-y}dy = \frac{1}{4}e^{-|x|}(1+|x|), \quad 0<|x|<\infty$

c) $E(X)=0$ puisque la fonction sous l'integrale est 'impair'.

2. #17 p271 Toutes les possibilités $(X_{i_1} < X_{i_2} < X_{i_3})$ sont equiprobables. Il y en a 6 dont 2 contiennent X_2 au centre $\Rightarrow$ prob. $= 2/6$

(1)

3. #19 p271 a) $f_Y(y) = \int_y^1 \frac{dx}{x} = -\ln y \quad , 0<y<1$

b) $f_X(x) = \int_0^x \frac{1}{x}\,dy = 1 \qquad 0<x<1 \qquad$ c) $EX = 1/2$

(4) d) $E(Y) = -\int_0^1 y\ln y\, dy \underset{\text{en partie}}{=} 1 + \int_0^1 (y\ln y - y)dy$

$\Rightarrow -2\int_0^1 y \ln y\, dy = 1 - \int_0^1 y\, dy = 1/2$

$\therefore EY = 1/4$

4. #21 p271 a) $\int_0^1\int_0^{1-y} 24xy\, dx\,dy = 1$

(3) b) $E(X) = \int_0^1\int_0^{1-y} 24x^2y\,dx\,dy = 2/5$

c) $E(Y) = \int_0^1\int_{x+y<1} 24xy^2\,dx\,dy = 2/5$

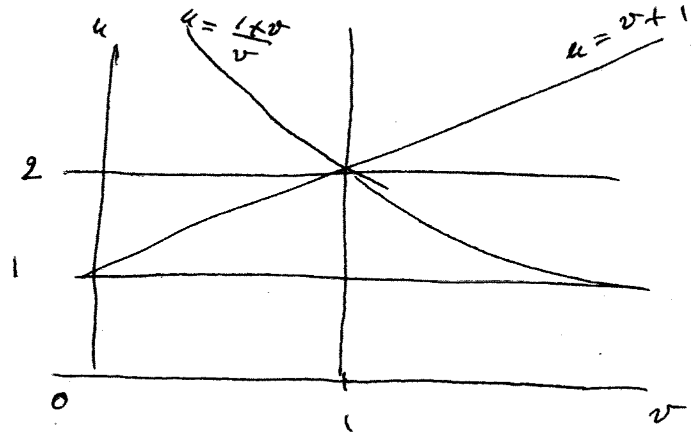
5. #56 p274 a)

$u = x+y, \quad v = x/y.$

$$f_{UV}(u,v) = \frac{u}{(1+v)^2},$$

(3) $0 < uv < 1+v$

$0 < u < 1+v$



b) $u = x, \quad v = x/y$

$$J = \begin{vmatrix} 1 & 0 \\ 1/y & -x/y^2 \end{vmatrix} = v^2/u$$

$$f_{UV}(u,v) = \frac{1}{|J|} = u/v^2, \quad 0 < u < 1, \quad 0 < u/v < 1$$

c) $u = x+y, \quad v = x/(x+y) \Rightarrow x = uv, \quad y = u(1-v)$

$$|J| = \frac{1}{x+y}$$

$$f_{U,V}(u,v) = x+y = u, \qquad 0 < u < 1/v$$